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Chapter 7 System of Equations

Section 7.5: Using Elimination to Solve system of Linear Equations

Here is another way to solving linear systems using algebra. Once again, it involves eliminating one of the variables…but in a different fashion than the last section!

Linear systems have two basic properties that we will need to know before we can learn this new way of solving system of linear equations.

Recall from Section 7.2 the question… Solve the following system graphically. Make a table of values

$y = -2x + 8$ → [1]

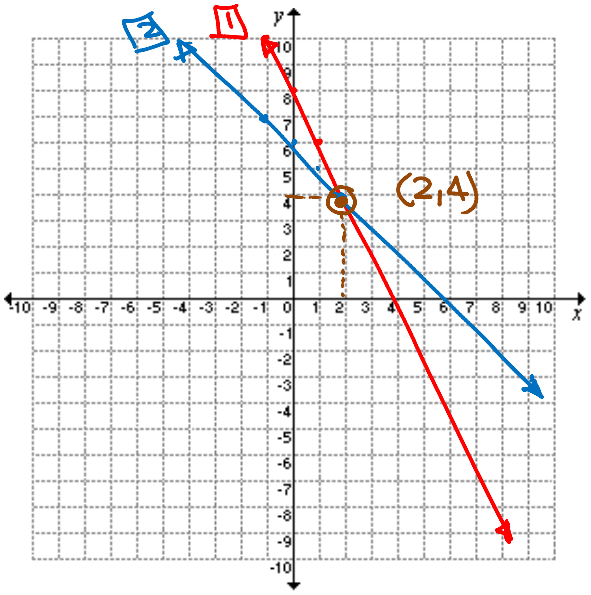
$x + y = 6$ → [2]

$Y = -2x + 8$

x	y
-1	10
0	8
1	6

$Y = 6 - x$

x	y
-1	7
0	6
1	5



Solution: (x,y) (2,4)

Investigation #1: Consider what happens if we multiply both sides of the first equation by 2 and both sides of the second equation by 3. Write out the new equations and make a table of values using the same x-values as above. Also graph it out…

$Y = -2x + 8$ → x2

$x + y = 6$ → x3

[1]

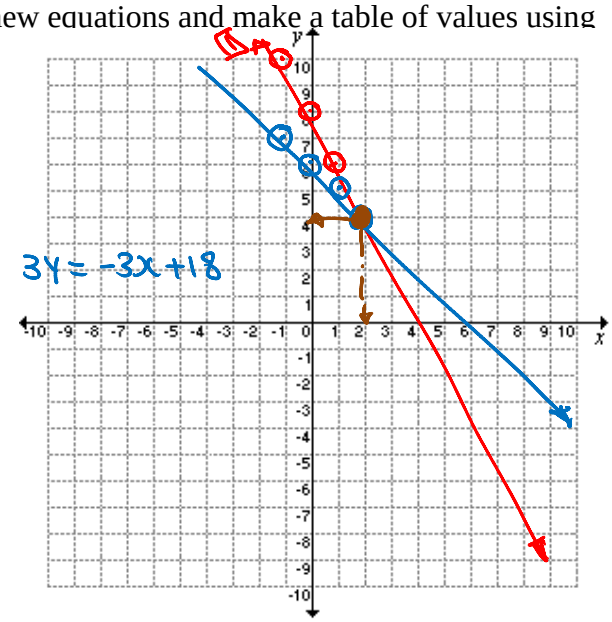
$2y = -4x + 16$

x	y
-1	10
0	8
1	6

[2]

$3x + 3y = 18$

x	y
-1	7
0	6
1	5



What do you notice?

Property #1: When you multiply both sides of either equation of a linear system by a constant,

does not change the solution because the linear systems that are formed are equivalent

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Investigation #2 Using the same two equations from the previous page $\begin{cases} y = -2x + 8 \\ x + y = 6 \end{cases}$, recall that the solution of this system was ~~(2,4)~~.

Let's move all the variables to the left side and consider what happens if we ADD the two equations…

$y = -2x + 8$ → $y + 2x = 8$ → [1]

$x + y = 6$ → $y + x = 6$ → [2]

different Eq. $2y + 3x = 14$ → [3]

Consider what happens if we SUBTRACT the two equations….

subtract $y + 2x = 8$ → [1] − $y + x = 6$ → [2]

$x = 2$

sub $x = 2$ in Eq. #[2]

$y + 2 = 6$

$y = 4$

solution (2,4)

Graph them out on the grid to the right and see what happens to the solution.

> Property #2: When you add or subtract equations of a linear system,
>
> ***Produces equivalent linear systems.***

Example 1: Solve the following systems by using the method of elimination.

a) $\begin{cases} 4x - 3y = 6 \\ 2x + 3y = 12 \end{cases}$ Add

$6x = 18$

$\frac{6x}{6} = \frac{18}{6}$

$x = 3$

Sub $x = 3$ in Eq. [1]

$4(3) - 3y = 6$

$12 - 3y = 6$

$-3y = 6 - 12$

$-3y = -6$

$y = 2$

(3,2)

b) $\begin{cases} x - 2y = 7 \\ 3x + 4y = 1 \end{cases}$ → [1], → [2]

multiply Eq. #[1] by ×2

$2x - 4y = 14$ → [1]

$3x + 4y = 1$ → [2] Add

$5x = 15$

$\frac{5x}{5} = \frac{15}{5}$

$x = 3$

sub. $x = 3$ in Eq. #[1]

$3 - 2y = 7$

$-2y = 4$

$y = -2$

solution (3,−2)

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Example 2: Solve the following systems by using the method of elimination.

a) $3x+2y=12$ ×2
$2x+3y=13$ ×3

$6x+4y=24$ → [1]
$-\ 6x+9y=39$ → [2]

$-5y=-15$

$\frac{-5y}{-5}=\frac{-15}{-5}$

$y=3$ ⟹ sub $y=3$ in #(2)

$2x+3(3)=13$
$2x+9=13$
$2x=4$
$x=2$
Solution $(2,3)$

b) $2x-3y=10$ ×2
$-4x+6y=-20$ → [2]
$4x-6y=20$ → [1]

$0+0=0$ true

infinity of solutions

A $(-3,6)$
B $(3,-6)$
C no solution
D infinit of solutions

#2 $-4x+6y=-20(x-1)$
$4x-6y=20$
same as Eq. # [1]

Example 3: Solve the following systems by using the method of elimination. You may need to simplify or rearrange the equations first!

a) $y+2x=10+4y$ → [1]
$4(x+y)=42-y$ → [2]

$-4y+y+2x=10$
$2x-3y=10$ → [1]

$4x+4y=42-y$
$4x+5y=42$ → [2]
$-\ 4x-6y=20$ → [1]''

×2

$11y=22$

$\frac{11y}{11}=\frac{22}{11}$

$y=2$

sub. $y=2$ in Eq. # [1]'

$2x-3(2)=10$
$2x-6=10$
$2x=16$
$x=8$
Solution $(8,2)$

Check in Eq. # [2]
$4(8+2)=42-2$
$40=40$ ✓

b) $\frac{x}{5}+\frac{3y}{10}=2$ → [1] $2x+3y=20$ → [1] ×4
$\frac{x}{2}+\frac{2y}{5}=\frac{29}{10}$ → [2] $5x+4y=29$ → [2] ×3

Eliminate y

$8x+12y=80$ → [1]
$-\ 15x+12y=87$ → [2]

$\frac{-7x}{-7}=\frac{-7}{-7}$

$x=1$ sub $x=1$ in Eq # [1]

$2(1)+3y=20$
$2+3y=20$
$3y=18$
$y=6$

Solution $(1,6)$

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Example 4: For the following word problems, create a system of linear equations and solve using the method of elimination.

a) A scientist worked for 6 days on a project and her assistant worked for 5 days, earning a total of $780 together. They earned $500 together on their next project, which required 4 days' work from the scientist and 3 days' work from the assistant. How much did each earn per day?

let (S) be the amount of money scientist earn per day
let (A) ~ ~ ~ ~ ~ Assistant ~ ~ ~

$6S + 5A = 780$ → [1] x2
$4S + 3A = 500$ → [2] x3

Eliminate S

$-\ 12S + 10A = 1560$ → [1]
$-\ 12S + 9A = 1500$ → [2]

$A = 60$

Assistant $60/day

Sub $A = 60$ in Eq. #[1]

$6S + 5(60) = 780$
$6S + 300 = 780$
$6S = 480$
$S = \$80/day$

b) A lifeguard earns an hourly rate for 20 h work in one week and an increased rate for overtime. One week Theresa worked 24 h and received $166.40. The next week she worked 27.5 h and received $200.00. Find her hourly rate and her overtime rate of pay.

let x be the hourly rate
let y be the overtime rate

$-\ 20x + 4y = 166.40$ → [1]
$-\ 20x + 7.5y = 200.00$ → [2]

$\frac{-3.5y}{-3.5} = \frac{-33.60}{-3.5}$

$y = \$9.60/hr$ For over time

Sub $y = 9.60$ in [1]

$20x + 4(9.60) = 166.40$
$20x + 38.40 = 166.40$
$20x = 128$
$x = \$6.40/hr$ hourly rate

(x, y) (6.40, 9.60)

Assignment: Page 437: 3, 4, 5, 8, 10, 12a, 13, 17, 18 &20